Fast Numerical Solution of Boltzmann Equation

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This talk consider a fast numerical solution of the classical Boltzmann equation for a simple, dilute gas of particles:

$$f_t + (v, \mathrm{grad}_x f) = \int\limits_{\mathbb{R}^3} \int\limits_{S^2} B(v,w,e)\,(f(v')f(w') - f(v)f(w))\; de\,dw\,,$$

where

$$f(t,x,v)\ :\ \mathbb{R}_+ \times \Omega \times \mathbb{R}^3 \to \mathbb{R}_+$$

describes the time evolution of the particle density.
It is observed that the particle density function can be considered as low multidimensional rank function and reads in the form

$$f(t,x,v) = \sum_{i_1,i_2,i_3}^{r_1,r_2,r_3} f^{(1)}_{i_1}(t,x,v_1) f^{(2)}_{i_2}(t,x,v_2) f^{(3)}_{i_3}(t,x,v_3) g_{i_1,i_2,i_3},$$

where r_1, r_2, r_3 are almost constants.
This approach needs $\mathcal{O}(n)$ words of memory for each grid points of domain discretisation, which is comparable to descretisation of Navier-Stokes equations, but produce better numerical accuracy. Here n denotes the number of discretisation points in one direction of the velocity space.

Many industrial examples, together with implementations on high performance hardware: distributed memory computers and GPUs are discussed during this presentation.